Debt and Insolvency

J.B. Heaton[*]

This Draft: January 2, 2019
Forthcoming, *Review of Banking and Financial Law*

Abstract

Legal rules play a powerful but understudied role in security design. This article presents two new theoretical results about the design of debt contracts. The results derive from the premise that firms must avoid legal insolvency when issuing new debt because insolvency at issuance would trigger severe operational limitations on the issuer. I first show that legal insolvency limits debt capacity, limiting the amount of money that a firm can raise with debt. I next show that legal insolvency limits the yield that firms can promise new creditors, helping to explain why there is no market for ultra-high-yield new issue debt. The article's results demonstrate that legal insolvency probably has important—if largely unnoticed—effects in limiting debt capacity and debt yields. More generally, the article illustrates the usefulness of better understanding legal solvency requirements, a legal mechanism that has received too little attention in the understanding of debt and security design.

I.   *Introduction*

Legal rules influence security design, but surprisingly little research addresses the role that legal rules play. With respect to the design of debt contracts, legal scholars have focused too

---

[*] J.B. Heaton Research LLC, jb@jbheaton.com.

narrowly on the role debt can play in disciplining managers,[1] a fact first discovered by financial economists.[2] Financial economists—too often ignorant of the legal rules that govern the behavior they purport to study—generate fanciful and complex models of security design with no apparent real world relevance.[3]

In this article, I explore two theoretical results about debt and the firm that take seriously one important legal mechanism: legal insolvency. I derive these results from the premise that firms must avoid legal insolvency when issuing new debt. Part II discusses the solvency tests in bankruptcy and corporate law that determine legal insolvency. Firms must avoid legal insolvency when issuing new debt because insolvency at issuance would trigger severe operational limitations, including creditor enforcement of fiduciary duties against the board of directors.[4] Some United States corporate law, including Delaware law that governs a disproportionately large number of United States corporations, allows creditors of a legally-insolvent corporation to enforce fiduciary duties against the board of directors,[5] while the corporate law of other commercially-important states, including that of New York, requires creditors to preserve the assets of an insolvent firm for the benefit of creditors.[6] Insolvency at debt issuance would also allow legal challenges to director and officer decisions that otherwise would be protected by the business judgment rule, most notably under fraudulent transfer laws that require the unwinding of transactions like corporate spin-offs[7] or leveraged buyouts,[8] the payment of dividends to shareholders,[9] or share

---

[1] *See, e.g.*, George G. Triantis, *Debt Financing, Corporate Decision Making, and Security Design*, 26 CAN. BUS. L.J. 93, 94-97 (1996) ("The fixed obligations of debt are a source of discipline on managers that requires minimal monitoring.").

[2] *See generally* Michael C. Jensen & William H. Meckling, *Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure*, 3 J. FIN. ECON. 305 (1976).

[3] There is a great deal of literature pertaining to modeling security design that do not account for legal rules. *See generally* Franklin Allen & Douglas Gale, *Optimal Security Design*, 1 REV. FIN. STUDIES 229 (1988); Arnoud W. A. Boot & Anjan V. Thakor, *Security Design*, 48 J. FIN. 1349 (1993); Peter M. DeMarzo & Yuliy Sannikov, *Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model*, 61 J. FIN. 2681 (2006); Mark Garmaise, *Rational Beliefs and Security Design*, 14 REV. FIN. STUDIES 1183 (2001); Milton Harris & Artur Raviv, *The Role of Games in Security Design*, 8 REV. FIN. STUDIES 327 (1995); Rafeal Repullo & Javier Suarez, *Monitoring, Liquidation, and Security Design*, 11 REV. FIN. STUDIES 163 (1998); Jaime F. Zender, *Optimal Financial Instruments*, 46 J. FIN. 1645 (1991). *But see* Ronald W. Anderson & Suresh Sundaresan, *Design and Valuation of Debt Contracts*, REV. FIN. STUDIES 37 (1996) (modeling the design of debt contracts with consideration of the role played by bankruptcy law).

[4] Quadrant Structured Prods. Co., Ltd. v. Vertin, 115 A.3d 535, 544 (Del. Ch. 2015) ("[T]he creditors of an insolvent corporation have standing to maintain derivative claims against directors on behalf of the corporation for breach of fiduciary duties." (quoting N. Am. Catholic Educ. Programming Found., Inc. v. Gheewalla 930 A.2d 92, 101 (Del. 2007))).

[5] *See Vertin*, 115 A.3d at 554.
("In my view, therefore, to maintain standing to sue derivatively, a creditor must establish that the corporation was insolvent at the time the creditor filed suit. The creditor need not demonstrate that the corporation continued to be insolvent until the date of judgment.").

[6] Pappas v. Gucciardo (*In re* Gucciardo), 577 B.R. 23, 34 (Bankr. E.D.N.Y. 2017) ("[D]irectors of an insolvent corporation owe a fiduciary duty to preserve the assets of the corporation for the benefit of the creditors." (quoting Econ. Dev. Growth Enters. Corp. v. McDermott, 478 B.R. 123, 128 (Bankr. N.D.N.Y. 2012))).

[7] *See generally* McGee Corp. v. Tronox (*In re* Tronox Inc.), 503 B.R. 239 (Bankr. S.D.N.Y. 2013) (discussing corporate spin-off that left entity insolvent and was fraudulent conveyance).

[8] *See* Wieboldt Stores, Inc. v. Schottenstein, 131 B.R. 655, 661 (N.D. Ill. 1991) (acknowledging payments made to selling shareholders in leveraged buyout could constitute fraudulent transfers).

[9] *See* DEL. CODE ANN. tit. 8, §§ 160 (2010); DEL. CODE ANN. tit. 8, §§ 160 (1998) (prohibiting payment of dividends while the corporation is insolvent or which will render it insolvent).

repurchases, and even the payment of salaries and bonuses or loans that are deemed too large to be of reasonably-equivalent benefit to the firm.[10]

Part III proves the basic results in a simple binary-outcome model of an issuing firm. I first show that the requirements of legal solvency tests constrain debt capacity. Without the need to satisfy solvency tests, firms could set face amounts of debts that far exceeded their expected ability to pay and the market value of their assets. While this debt would of course not issue at par (that is, the firm could only raise a likely heavily-discounted amount of the issued face value), the ability to issue debt at high face value would increase the amount of debt proceeds the firm could raise in the capital market. But legal solvency tests constrain this outcome, explaining why the face amount of debt is as low as we see in the real capital markets.

I next show that legal solvency tests also, in a related way, constrain the yield that issuers can offer on their debt, helping to explain why there is no market for ultra-high-yield new issue debt. While much corporate debt eventually trades at very high yields because of increasing probability of default and low recovery, ultra-high-yield new issue debt is, with the exception of some very specialized structured notes, absent in the corporate market. For example, on May 31, 2018, the private placement of $650 million of senior unsecured bonds of PetSmart Inc. issued one year earlier at par with a coupon of 8.875% had a yield to maturity (mid of bid/ask) of over 25%,[11] while in the first five full months of 2018 the highest yield at issue date for a bond reported on Bloomberg's extensive database of United States issuers with yield at issue data was about 11.9% for the private placement of $1.3 billion senior unsecured bonds of McDermott Technologies Americas Inc. issued April 18, 2018.[12] This is hard to explain because ultra-high-yield debt would allow an issuer to offer a high return to investors (say 25% or 35% per year), far above the yield on typical high-yield bonds, but far below the returns on convertible debt that is converted to equity when the company is successful.[13] At the same time, ultra-high-yield debt would offer debt-security purchasers an opportunity for higher returns without sacrificing the contractual and other creditors-rights protections of a debt instrument. Nevertheless, the market for ultra-high-yield debt is missing. The need to avoid legal insolvency *at issuance* may explain why.

Part IV shows that the results are not dependent on the simple modeling framework used in Part II, but generalize to the best available model of the firm, the Black-Scholes option pricing

---

[10] *See* Acequia, Inc. v. Clinton (*In re* Acequia, Inc.), 34 F.3d 800, 805–06 (9th Cir. 1994) (finding amounts that corporate founder claimed were personal loans or reimbursements were avoidable fraudulent transfers made by corporation).

[11] CUSIP: 716768AD8, data from Bloomberg accessed May 31, 2018. The debt was originally issued on May 31, 2017.

[12] CUSIP: 58003XAA00, data from Bloomberg accessed May 31, 2018. The debt matures on May 1, 2024, has a coupon of 10.625%, and was issued at 94.75. McDermott Escrow 1, Inc., Merger Agreement Exhibit 4.1 (Form 8-K, Exhibit 4.1) (Apr. 18, 2018). In a study by Paul Asquith, David W. Mullins, and Eric D. Wolff, the highest average yield to maturity for new issue high-yield debt was only 17.395% for the year 1981 when the 10-year Treasury rate exceeded 12%. *Original Issue High Yield Bonds: Aging Analyses of Defaults, Exchanges, and Calls*, 44 J. FIN. 923 (1989).

[13] An explanation that fails is usury laws. Corporate borrowers are generally unable to assert the defense of usury. *See, e.g.*, N.Y. GEN. OBLIG. LAW § 5-501 (McKinney 2018) ("No law regulating the maximum rate of interest which may be charged, taken or received, including . . . the penal law, shall apply to any loan or forbearance in the amount of two million five hundred thousand dollars or more."); N.Y. GEN. OBLIG. LAW § 5-521 (McKinney 2018) ("No corporation shall hereafter interpose the defense of usury in any action. The term corporation, as used in this section, shall be construed to include all associations, and joint-stock companies having any of the powers and privileges of corporations not possessed by individuals or partnerships.").

model. Indeed, the results are easier to derive there, but intuition is aided by the simple binary-outcome approach in Part II.

A short conclusion follows Part III.

II.   *Solvency Tests in Bankruptcy and Corporate Law*

Two solvency tests appear in bankruptcy and corporate law.[14] First is a test of whether a firm can be expected to pay its debts as they come due (the ability-to-pay solvency test, sometimes referred to as cash-flow solvency or equitable solvency).[15] The ability-to-pay solvency test asks if the firm can reasonably expect to pay its debts as they come due. In the federal fraudulent conveyance statute, for example, the question is whether the firm "intended to incur, or believed that [it] would incur, debts that would be beyond [its] ability to pay as such debts matured."[16] The Uniform Fraudulent Transfer Act states the test similarly, asking whether the transferor "intended to incur, or believed or reasonably should have believed that the debtor would incur, debts beyond the debtor's ability to pay as they became due."[17] The Uniform Fraudulent Conveyance Act, still the law in New York, asks whether the debtor "intends or believes that he will incur debts beyond his ability to pay as they mature."[18] The ability-to-pay test is a forward-looking test: it is not enough to be able to meet current obligations; the firm must be able to meet its future obligations as well.[19]

---

[14] For detailed discussion of legal solvency tests, see Lyondell Chem. Co. v. Blavatnik (*In re* Lyondell Chem. Co.), 567 B.R. 55, 108–09 (Bankr. S.D.N.Y. 2017), *aff'd*, 585 B.R. 41 (S.D.N.Y. 2018); J.B. Heaton, *Solvency Tests*, 62 Bus. Law. 983 (2007). A third test, for unreasonable small capital, is also used, *see In re* Lyondell Chem. Co., 585 B.R. at 83 ("As discussed in detail below, the three financial condition tests are: (i) balance-sheet insolvency, (ii) unreasonably small capital, and (iii) the intent to incur debts beyond the debtor's ability to pay the debts as they come due."); Heaton, *supra*, at 995–996 (discussing this third test at length, which is essentially only a test of whether the entity is "close" to being insolvent under one of the other two measures). *See also In re* AWTR Liquidation Inc., 548 B.R. 300, 312 (Bankr. C.D. Cal. 2016) ("The second and third insolvency tests described above—inadequate capital and cash flow/equitable insolvency—may be seen as different iterations of the same test: inability to pay debts either in the reasonably foreseeable future or more immediately.").

[15] *See, e.g.*, Sec. Inv'r Prot. Corp. v. Glob. Arena Capital Corp., 164 F. Supp. 3d 531, 537–38 (S.D.N.Y. 2016) (stating that the ability-to-pay test "equates insolvency with a lack of liquid funds, or the inability to pay one's debts in the ordinary course of business as the debts mature" (quoting Poseidon Pool & Spa Recreational, Inc. (*In re* Poseidon), 443 B.R. 271, 280 (E.D.N.Y. 2010))).

[16] 11 U.S.C. § 548 (a)(1)(B)(ii)(III) (2012).

[17] *See, e.g.*, the Uniform Fraudulent Transfer Act as adopted in Illinois. 740 ILL. COMP. STAT. ANN. 160/5(a)(2)(B) (West 2018) ("A transfer made or obligation incurred by a debtor is fraudulent as to a creditor, whether the creditor's claim arose before or after the transfer was made or the obligation was incurred, if the debtor made the transfer or incurred the obligation: . . . (2) without receiving a reasonably equivalent value in exchange for the transfer or obligation, and the debtor: . . . (B) intended to incur, or believed or reasonably should have believed that he would incur, debts beyond his ability to pay as they became due.").

[18] *See* N.Y. DEBT. & CRED. LAW § 275 (McKinney 2018) ("Every conveyance made and every obligation incurred without fair consideration when the person making the conveyance or entering into the obligation intends or believes that he will incur debts beyond his ability to pay as they mature, is fraudulent as to both present and future creditors.").

[19] *See, e.g.*, Telegobe USA, Inc. v. BCE Inc. (*In re* Teleglobe Commc'ns Corp.), 392 B.R. 561, 602 (Bankr. D. Del. 2008) ("The Court agrees with the Defendants. The cash flow test is 'forward looking' in the sense that '[i]t is not enough to be able to meet current obligations; the firm must be able to meet its future obligations as well.'" (quoting Heaton, *supra* note 14, at 989)).

The second solvency test asks a different question than the ability-to-pay test: is the market value of this firm's assets greater than the face value of its liabilities?[20] The United States Bankruptcy Code defines the word "insolvent" as the "financial condition such that the sum of such entity's debts is greater than all of such entity's property, at a fair valuation."[21] Fair valuation, in practice, means the application of standard valuation methods, including discounted cash flow and multiples-based valuation for going-concern businesses.[22] Debts are valued at face value,[23] contingent liabilities are discounted for their probability of occurrence,[24] and, of course, a positive accounting book balance determined under the Generally Accepted Accounting Principles (GAAP) does not imply legal solvency.[25] The line between solvency and insolvency has important consequences for firms. When a corporation is solvent, Delaware corporate law requires that directors maximize the long-term benefit of the corporation to the corporation's shareholders,[26] and allows directors to pay out corporate assets to shareholders in the form of dividends and repurchases.[27]

---

[20] *See*, *e.g.*, Fisher v. Pa. State Univ. (*In re* Fisher), 575 B.R. 640, 645 (Bankr. M.D. Pa. 2017) ("Bankruptcy courts generally use a balance sheet test for insolvency; comparing assets to liabilities. This does not always involve a literal examination of the debtor's balance sheet; because, for example, the value of a company's assets may need to be adjusted up or down to reflect its going concern value.").

[21] 11 U.S.C. § 101(32)(A) (2012); French v. Nardolillo (*In re* Perry), 158 B.R. 694, 697 (Bankr. N.D. Ohio 1993) ("This definition of insolvency provides the Court with a 'balance sheet' test to determine insolvency.").

[22] *See*, *e.g.*, *In re* SunEdison, Inc., 556 B.R. 94, 104 (Bankr. S.D.N.Y. 2016) ("Fair value, in the context of a going concern, is determined by the fair market price of the debtor's assets that could be obtained if sold in a prudent manner within a reasonable period of time to pay the debtor's debts. Balance sheets, on the other hand, reflect book value, which does not ordinarily equate to market value.") (internal citations and quotation marks omitted).

[23] *See, e.g.*, Travellers Int'l v. Trans World Airlines, Inc. (*In re* Trans World Airlines, Inc.), 134 F.3d 188, 197 n.7 (3d Cir. 1998) ("If holders of claims are fully informed of the debtor's affairs and the asset values are less than the face amount of the claims, they would never value their claims at more than the value of the assets. Likewise, the fully informed debtor would never be willing to pay claimants more than claimants would be willing to take. Thus, the value of the claims would never exceed the value of the assets and insolvency could never occur.").

[24] *See, e.g.*, *In re* Xonics Photochemical, Inc., 841 F.2d 198, 200 (7th Cir. 1988) (holding that courts must take into account the contingent value of assets when considering their value).

[25] *See*, *e.g.*, Lids Corp. v. Marathon Inv. Partners, L.P. (*In re* Lids Corp.), 281 B.R. 535, 540 (Bankr. D. Del. 2002) ("This standard for solvency is typically called the 'Balance Sheet Test.' However, this may be a misnomer because the Balance Sheet Test is based on a fair valuation and not based on [GAAP], which are used to prepare a typical balance sheet." (citation omitted)); F & S Mfg. Corp v. Buildex, Inc. (*In re* Estate of F & S Cent. Mfg. Corp.), 53 B.R. 842, 849 (Bankr. E.D.N.Y. 1985) ("Asset values carried on a balance sheet, even if derived in accordance with 'generally accepted accounting principles,' do not necessarily reflect fair value: 'Generally accepted accounting principles' are not synonymous with any specific [valuation] policy.") (internal citations and quotation marks omitted).

[26] *See*, *e.g.*, *In re* Rural Metro Corp., 88 A.3d 54, 80 (Del. Ch. 2014), *clarified on denial of reargument sub nom.*, *In re* Rural Metro Corp. Stockholders Litig., No. 6350-VCL, 2014 WL 1094173 (Del. Ch. Mar. 19, 2014) ("More concretely, the fiduciary relationship . . . required that the directors act prudently, loyally, and in good faith to maximize [the company's] value over the long-term for the benefit of its stockholders.").

[27] There are few restrictions on the payment of dividends or the making of share repurchases under typical corporate law in the United States when the firm is solvent. Under Delaware law, the corporation's directors can declare and pay dividends out of its surplus (the amount by which net assets exceed liabilities and the corporation's stated capital). DEL. CODE ANN. tit. 8, § 154 (2010). If there is no such surplus, the corporation's directors may declare and pay dividends "out of its net profits for the fiscal year in which the dividend is declared and/or the preceding fiscal year." DEL. CODE ANN. tit. 8, § 170(a)(2) (2010). This is true only if the capital has not been impaired, that is, fallen below the amount of the preference of any stock with preference rights. *Id.* The directors of a Delaware corporation may cause the corporation to purchase the corporation's own shares, again, so long as the corporation's capital is not impaired. DEL. CODE ANN. tit. 8, § 160 (2010).

When the firm is solvent, many of the directors' decisions receive the protection of the business judgment rule.[28] When the corporation is insolvent, however, it loses its ability to make gratuitous asset transfers and to avoid the second guessing of its decisions because fraudulent transfer law prevents transfers for less than approximately equivalent value[29] and dividends and share repurchases, for example, have been determined not to give a benefit to the corporation, only to the corporation's stockholders.[30] Under Delaware law, when the corporation is insolvent, its *creditors* can sue to enforce the board of directors' fiduciary duties to the corporation, something they cannot do when the corporation is solvent.[31] "Under New York law, directors of an insolvent corporation owe a fiduciary duty to preserve the assets of the corporation for the benefit of creditors."[32] California law is similar.[33] Moreover, insolvency is a "badge of fraud" that a litigant suing to undo a corporate asset transfer can offer as evidence that the asset transfer (including dividends or a spin-off) was intentionally fraudulent toward creditors.[34] The law of some states, such as Illinois,[35] Florida,[36] and Arizona,[37] even provide for a shift of enforcement of fiduciary duties to creditors that are only in the vicinity of insolvency and not yet actually insolvent.

Put simply, there are things a solvent firm may do—such as pay dividends—that an insolvent firm may not. From an economic perspective, limitations on an insolvent firm's activities increase debt capacity *ex ante* by protecting creditors *ex post* from actions that make it

---

[28] *See*, *e.g.*, Aronson v. Lewis, 473 A.2d 805, 812 (Del. 1984) ("The business judgment rule is an acknowledgment of the managerial prerogatives of Delaware directors . . . . It is a presumption that in making a business decision the directors of a corporation acted on an informed basis, in good faith and in the honest belief that the action taken was in the best interests of the company.") (citation omitted); Erie Cty. Emp. Ret. Sys. v. Blitzer (*In re* Kenneth Cole Prods., Inc.), 52 N.E.3d 214, 218 (N.Y. 2016) ("We begin with the general principle that courts should strive to avoid interfering with the internal management of business corporations. To that end, we have long adhered to the business judgment rule, which provides that, where corporate officers or directors exercise unbiased judgment in determining that certain actions will promote the corporation's interests, courts will defer to those determinations if they were made in good faith.").

[29] *See* 11 U.S.C. § 548 (2012).

[30] *See*, *e.g.*, Feinberg v. RM Acquisition, LLC, 629 F.3d 671, 674 (7th Cir. 2011) ("[A] dividend is not an exchange for reasonably equivalent value . . . .").

[31] *See*, *e.g.*, N. Am. Catholic Educ. Programming Found., Inc. v. Gheewalla, 930 A.2d 92, 101 (Del. 2007); Quadrant Structured Prods. Co., Ltd. v. Vertin, 115 A.3d 535, 554 (Del. Ch. 2015).

[32] Econ. Dev. Growth Enters. Corp. v. McDermott, 478 B.R. 123, 128 (Bankr. N.D.N.Y. 2012) (citation omitted).

[33] *See*, *e.g.*, Scouler & Co. v. Schwartz, No. 11-CV-06377 NC, 2012 WL 1502762, at *5 (N.D. Cal. Apr. 23, 2012) ("When a corporation becomes insolvent, California's trust fund doctrine imposes an additional, albeit limited, fiduciary duty on the corporation's directors to avoid 'actions that divert, dissipate, or risk corporate assets that might otherwise be used to pay creditors' claims,' including 'acts that involve self-dealing or the preferential treatment of creditors.' This additional duty is owed to the corporation's creditors only . . . ." (citation omitted)).

[34] For example, the Uniform Fraudulent Transfer Act as adopted in Illinois states that "[i]n determining actual intent [to defraud creditors] . . . consideration may be given, among other factors, to whether . . . the debtor was insolvent or became insolvent shortly after the transfer was made or the obligation was incurred . . . ." 740 ILL. COMP. STAT. ANN. 160/5(b) (West 2018).

[35] *See*, *e.g.*, Bachrach Clothing, Inc. v. Bachrach (*In re* Bachrach Clothing, Inc.), 480 B.R. 820, 828 (Bankr. N.D. Ill. 2012) ("Generally, directors owe fiduciary duties only to their shareholders. However, when a company is operating in the zone of insolvency, Illinois law expands that duty to the company's creditors.").

[36] *See*, *e.g.*, Brown v. Luboff (*In re* Sigma-Tech Sales, Inc.), 570 B.R. 408, 418 (Bankr. S.D. Fla. 2017) ("An officer's or director's fiduciary duties are extended to the creditors of a corporation when the corporation becomes insolvent or is in the 'vicinity of insolvency.'" (quoting Welt v. Jacobson (*In re* Aqua Clear Techs., Inc.), 361 B.R. 567, 575 (Bankr. S.D. Fla. 2007))).

[37] *See*, *e.g.*, Dooley v. O'Brien, 244 P.3d 586, 591 (Ariz. Ct. App. 2010) ("[T]he duties of a director or officer of a corporation are implied by law. Indeed, these fiduciary obligations can apply even to creditors when a corporation enters the zone of insolvency, without regard to the terms in the underlying contracts." (citing Dawson v. Withycombe, 163 P.3d 1034, 1057 (Ariz. Ct. App. 2007))).

less likely that the firm will pay its debts.[38] Creditors who know that such limits are in place *ex post* are willing to lend more money to the firm *ex ante*.[39] Firms would have little or no debt capacity if there were no controls on their power to intentionally reduce their ability to pay debts by transferring assets after they have borrowed money.[40]

In practice, however, solvency testing is fraught with difficulties. Consider the ability-to-pay test, for example, and a firm that owes a debt requiring a near-term payment of $100 but has no assets or cash on hand to make that payment. Further, assume that before the payment is due the firm either will receive $1000 with a 15% probability, or will receive $0 with an 85% probability. There is an 85% probability that the firm will not pay its debt when it comes due and only a 15% probability that the firm will pay its debts when they come due. If the 15% likelihood comes to pass, and the firm receives $1000, then it will pay off the $100 debt leaving $900 for the firm's owners. The expected cash flow, nevertheless, is $150 because 15% of $1000 is $150. A firm can have an expected cash flow that is higher than its debt even when it is almost sure (in the example, 85% sure) to be unable to pay its debt when it comes due. But if ability-to-pay solvency is not determined by expected cash flow, what measure of ability to pay does make sense? Nearly any corporation with debt has some chance of defaulting on that debt depending on what happens to it in the future. Some default risk is par for the course for virtually any debtor (leaving aside governments that may pay their obligations using their own currency). One must conclude that "ability to pay" means something less than an ability to pay with certainty, but more than a very low probability, such as 15% in the example. The case law provides little helpful guidance.

Similar difficulties—also unresolved in the case law—plague the balance-sheet test.[41] In applying the balance-sheet solvency test, courts struggle with how to value assets and, in particular, whether to value a firm's assets as if the firm continues as a going concern (that is, to assume the firm will continue in operation and generate cash flow from its business) or in liquidation (that is, as if the firm has ceased or will cease doing business and will dispose of its corporate property).[42] The typical approach is to determine first whether, in fact, the debtor was a going concern or was "on its deathbed" at a relevant date and then to value the assets according to that finding.[43] That is, in determining a "debtor's insolvency at the time of an alleged fraudulent transfer," the bankruptcy court must perform a "fact-intensive determination" based on its "review of the debtor's financial records and status at time of transfer."[44]

---

[38] *See* J.B. Heaton, *Incomplete Financial Contracts and Non-Contractual Legal Rules: The Case of Debt Capacity and Fraudulent Conveyance Law*, 9 J. Fin. Intermediation 169, 170–71 (2000) (arguing that debt capacities are limited when borrowers cannot commit to not fraudulently transferring assets).

[39] *See id.* at 176–79 (showing the effects that fraudulent conveyance law, which limits some firm activities, has on the availability of credit).

[40] *See generally id.*

[41] *See* Sherman v. FSC Realty LLC (*In re* Brentwood-Lexford Partners, LLC), 292 B.R. 255, 268 (Bankr. N.D. Tex. 2003) (citing *In re* DAK Indus. Inc., 170 F.3d 1197, 1199-1200 (9th Cir. 1999) and *In re* Taxman Clothing Co., Inc., 905 F.2d 166, 169–70 (7th Cir. 1990)) (applying the balance sheet test through several complex calculations).

[42] *See id.* at 268–72.

[43] *Id.* at 268.See, for example, Sherman v. FSC Realty LLC (In re Brentwood-Lexford Partners, LLC), 292 B.R. 255, 268 (Bankr. N.D. Tex. 2003), citing In re DAK Indus. Inc., 170 F.3d 1197, 1199-1200 (9th Cir. 1999) and In re Taxman Clothing Co., Inc., 905 F.2d 166, 169–70 (7th Cir. 1990)).

[44] Williams v. Wu (*In re* TTC Plaza Ltd. P'ship), No. 11-38381, 2014 WL 3057555, at *2 (Bankr. S.D. Tex. July 7, 2014) (citations and quotations omitted).

To perform the balance sheet insolvency test, courts conduct a two-step analysis. The court first determines whether the debtor was a "going concern" or was "on its deathbed." The court must then value the debtor's assets, depending on the status determined in the first inquiry, and apply the balance sheet test to determine whether the debtor was solvent. For a debtor that was a going concern, the court would "determine the fair market price of the debtor's assets as if they had been sold as a unit, in a prudent manner, and within a reasonable time." As a going concern, the debtor would not likely face a forced sale. Accordingly, a fair market valuation best determines a fair market price.[45]

The choice is typically difficult, however, because most litigated solvency tests involve an entity that was, at the test date, "midway between a prosperous going concern and a dead enterprise."[46] Such cases also can create an interesting anomaly. By demanding going concern valuation except where a company is about to close its doors, the courts force litigants to construct going concern valuations even where liquidation was the optimal decision as of the valuation date. As one judge put it about a century ago:

> The effort [in such valuations] is to find out not what a real buyer and a real seller, under the conditions actually surrounding them, do, but what a purely imaginary buyer will pay a make-believe seller, under conditions which do not exist. You are forced to wonder what would have happened if everything had been different from what it was. It is not easy to guess what will take place in Wonderland, as other people than Lewis Carroll's heroine have found out.[47]

If the firm's condition warrants a liquidation assumption, that determination also can define the speed with which such a hypothetical liquidation takes place.[48] Economic theory suggests that the sale price of an asset increases when additional time is spent marketing the asset,[49] and courts usually avoid assuming that assets are liquidated in fire-sale auctions presumably on the assumption that they are not value-maximizing. Where circumstances dictate, however, courts will allow the assumption of a fairly quick and harsh liquidation.[50]

Solvency determinations can, of course, differ depending on the solvency test that is applied. Solvency under the ability-to-pay test does not imply balance-sheet solvency, and *vice versa*.[51] Ability-to-pay and balance-sheet solvency tests measure different things about the

---

[45] *Id.*

[46] James C. Bonbright & Charles Pickett, *Valuation to Determine Solvency Under the Bankruptcy Act*, 29 COLUM. L. REV. 582, 582 (1929).

[47] McGill v. Commercial Credit Co., 243 Fed. 637, 647 (D. Md. 1917).

[48] *See TTC Plaza,* 2014 WL 3057555, at *3.

[49] *See, e.g.,* Steven A. Lippman & John J. McCall, *An Operational Measure of Liquidity*, 76 AMER. ECON. REV. 43, 50 (1986) ("Makower and Marschak describe their concept of saleability 'as the relationship between the selling price and the time which the seller must wait in order to get it'. . . . [T]hey state that 'the influence of time on the selling price is due to the seller's finding more buyers.'").

[50] *See, e.g.,* Osherow v. Texas Silica Logistics Joint Venture (*In re* FWLL, Inc.), No. 15-52071-CAG, 2018 WL 1684308, at *15 (Bankr. W.D. Tex. Apr. 5, 2018) ("The Court finds that the evidence showed that, at the time of the transfers, FWLL was insolvent, on its financial deathbed, and on the verge of filing bankruptcy."); Schwinn Plan Comm. v. AFS Cycle & Co., Ltd. (*In re* Schwinn Bicycle Co.), 192 B.R. 477, 487 (Bankr. N.D. Ill. 1996) (finding that Schwinn "was not financially viable, was in severe financial distress and was on its 'deathbed' when it filed for bankruptcy and during the prior ninety days.").

[51] Sec. Inv'r Prot. Corp. v. Glob. Arena Capital Corp., 164 F. Supp. 3d 531, 537–38 (S.D.N.Y. 2016) (differentiating the equity test of insolvency, which equates insolvency with the inability to pay one's debts in the ordinary course of

firm.[52] The ability-to-pay solvency test asks whether cash flow can be matched to the face value of the maturing debt obligations in the period that the debt obligation matures.[53] There is, in essence, no impact of the time value of money, nor are cash flows in a given period reduced in value because of the risk characteristics that investors might attach to them.[54] The test simply is whether cash is likely (where the likelihood necessary to pass the test is of course open to debate) to be sufficient to pay the face value of the debts when they mature.[55] Of course, as discussed below, the test allows assumptions about financing from assets to move cash flows from one period to another where possible.

By contrast, the balance-sheet test involves valuation.[56] In a going-concern valuation, future cash flows are discounted to present value to reflect a time value of money and the risk these cash flows have to an investor.[57] It is obvious that the "cash flow of a firm which might in future periods match maturing debt obligations could nevertheless be reduced to a sufficient degree (to reflect the time value of money and risk) such that the value of the firm will be below the face value of its debt."[58] "Thus, a firm that has the ability to pay its debts in all future periods under the ability-to-pay test may still be deemed insolvent under the going-concern balance-sheet test."[59] As for liquidation valuation, one possible liquidation value for the firm's assets is, of course, its going-concern value.[60] But liquidation values in particular industries may be low during times when firms are temporarily cash constrained.[61] The highest valuing buyers of the firm's assets in liquidation may be other firms in its industry, and because industries may experience downturns or credit constraints at the same time, liquidation values may be temporarily low even for a firm that may otherwise expect to pay its debts as they come due.[62] Any other restriction on the marketability of the firm's assets can also have a large effect on liquidation value.[63]

Just as ability-to-pay solvency does not imply balance-sheet solvency, the converse is true as well: balance-sheet solvency does not imply ability-to-pay solvency.[64] Ability-to-pay solvency requires that the firm be able to match cash in a particular period with the amount of debt maturing in that period.[65] A going-concern balance-sheet solvency test, however, requires only that the discounted sum of future expected cash flows exceed the sum or total amount of

---

business as the debts mature, with the bankruptcy test, which focuses on whether a firm's liabilities exceed its assets).

[52] *See id.*

[53] *See id.*

[54] *See id.*

[55] *Id.*

[56] *Id.*

[57] Alternatively, going-concern values are derived from comparable companies or comparable transactions. Heaton, *supra* note 14, at 997.

[58] *Id.*

[59] *Id.*

[60] *Id.* at 998.

[61] *See* Andrei Shleifer & Robert W. Vishny, *Liquidation Values and Debt Capacity: A Market Equilibrium Approach*, 47 J. FIN. 1343, 1346–55 (1992) (presenting a model where a firm in financial distress needs to sell assets at the same time its industry peers, the highest-valuing users, also are likely to be experiencing financial distress).

[62] *Id.* at 1346.

[63] *See* Francis A. Longstaff, *How Much Can Marketability Affect Security Values?*, 50 J. FIN. 1767, 1768 (1995) (showing theoretically how discounts for marketability can have large effects on value).

[64] Heaton, *supra* note 14, at 998.

[65] *Id.* at 988 ("The ability-to-pay solvency test asks if the firm can reasonably expect to pay its debts as they come due.").

debt at face value.[66] A going-concern balance-sheet solvency test does not concern itself with the possible mismatch between cash flows that make up the discounted sum and cash flows that must be paid to meet maturing debt obligations.[67] This can hide a liquidity problem.[68] "That is, a firm may be going-concern balance-sheet solvent but still unable to turn its going concern value into cash as needed at particular points in time to pay particular debts as they mature."[69] Thus, going-concern balance-sheet solvency test does not imply ability-to-pay solvency if the going concern value is illiquid.[70]

Recently, courts have placed greater emphasis on market evidence of solvency or insolvency.[71] This trend stems as much from skepticism about analytical valuation methods in the hands of paid expert witnesses than on a judicial understanding of market evidence.[72] But given the difficulty of cash flow projection and the many free parameters in, for example, a discounted cash flow analysis,[73] the use of market prices can be highly informative.[74] Of course, it is important in using market evidence to understand that, for example, the equity of an insolvent firm can trade at substantial values and its debt can trade at a meaningful but not deep discounts.[75]

III. *A Binary-Outcome Model of an Issuing Firm*

I start by developing the two main results in a simple binary-outcome (a "good"-state and "bad"-state) model of a debtor firm. The model demonstrates how debt issuance unconstrained by legal solvency tests is potentially larger in amount and higher in yield, then demonstrates the impact of legal insolvency avoidance on debt capacity and debt yields.

A. Set Up

---

[66] *Id.* at 991.

[67] *Id.* at 999.

[68] *Id.*

[69] *Id.*

[70] *See, e.g.*, Harry DeAngelo, Linda DeAngelo & Stuart C. Gilson, *The Collapse of First Executive Corporation: Junk Bonds, Adverse Publicity, and the 'Run On the Bank' Phenomenon*, 36 J. FIN. ECON. 287, 316–18 (1994) (arguing that First Executive Corporation may have been balance-sheet solvent even as it was unable to pay its debts as they came due).

[71] *See* Gregory A. Horowitz, *A Further Comment on the Complexities of Market Evidence in Valuation Litigation*, 68 BUS. L. 1071, 1074–76 (2013) (analyzing several bankruptcy courts' decisions regarding the relationship between the values of equity and debt); Robert J. Stark, Jack F. Williams & Anders J. Maxwell, *Market Evidence, Expert Opinion, and the Adjudicated Value of Distressed Businesses*, 68 BUS. L. 1039, 1040 (2013) (illustrating the more extensive judicial reliance on market evidence instead of market opinion).

[72] Michael W. Schwartz & David C. Bryan, *Campbell, Iridium, and the Future of Valuation Litigation*, 67 BUS. L. 939, 940 (2012).

[73] *See, e.g.*, Stuart Gilson, Edith Hotchkiss & Richard Ruback, *Valuation of Bankrupt Firms*, 13 REV. FIN. STUD. 43, 44–45 (2000) (finding large valuation errors in their study of discounted cash flow methods in bankruptcy); Steven N. Kaplan & Richard Ruback, *The Valuation of Cash Flow Forecasts: An Empirical Analysis*, 50 J. FIN. 1059, 1061 (1995) (documenting statistically significant upward bias of both operating income and operating margins in discounted cash flow analyses of leveraged buyouts and recapitalizations).

[74] *See* Kaplan, *supra* note 73, at 1039–1041(discussing the value of market data in determining a company's value).

[75] *See* J.B. Heaton, *Positive Equity Prices with Insolvency Under Legal Solvency Tests*, 23 J. FORENSIC ECON. 63, 68–69 (2018); J.B. Heaton, *Worthless Companies*, EUROPEAN FIN. MGMT., (forthcoming 2018) (on file with author).

Consider a simple binary-outcome model of a debtor firm. There are two periods, time $t = 0$ and time $t = 1$. Investors are risk neutral and value firms at the expected value of their possible future cash flows. The risk-free rate is zero. There are two possible states of the world at time $t = 1$, a good state with cash flow G and a bad state with cash flow B, where $G > B$. The probability of the good state is p_G and the probability of the bad state is $1 - p_G$. The value of the assets at time $t = 0$ is the probability-weighted sum of the good and bad cash flows:

$$V_{A,0} = p_G G + (1 - p_G)B$$

The firm has debt with a face value at time $t = 0$ of D that is due at time $t = 1$. Debt is entitled to full payment of D or the entire cash flow in the state, whichever is smaller. The value of the debt at time $t = 0$ is

$$V_{D,0} = p_G min\{G, D\} + (1 - p_G)min\{B, D\}$$

The firm's equity receives the amount, if any, remaining after the repayment of debt. The value of the equity at time $t = 0$ is

$$V_{E,0} = p_G max\{G - D, 0\} + (1 - p_G)max\{B - D, 0\}$$

I define balance-sheet solvency and ability-to-pay solvency as follows:
Balance-Sheet Solvency: The firm is *balance-sheet solvent* at time $t = 0$ if and only if
$$V_{A,0} \geq D$$
The firm is thus *balance-sheet insolvent* if $V_{A,0} < D$.

Ability-to-Pay Solvency: The debtor firm is *ability-to-pay solvent* at time $t = 0$ if and only if

$$p_G I_{G \geq D} + (1 - p_G)I_{B \geq D} \geq p_S$$

where $I_{G \geq D}$ is an indicator function taking the value of 1 if $G \geq D$ and 0 otherwise, $I_{B \geq D}$ is an indicator function taking the value of 1 if $B \geq D$ and 0 otherwise, and p_S is the minimum probability by which a firm must be expected to pay its debt D in full to be considered ability-to-pay solvent. The firm is *ability-to-pay insolvent* if $p_G I_{G \geq D} + (1 - p_G)I_{B \geq D} < p_S$.

B.   Solvency Can Differ Under the Two Tests

I first show that balance-sheet insolvency does not imply ability-to-pay insolvency and *vice versa*.

PROPOSITION 1: *Balance-sheet insolvency does not imply ability-to-pay insolvency. That is,* $V_{A,0} < D \nRightarrow p_G I_{G \geq D} + (1 - p_G)I_{B \geq D} < p_S$.

Proof: $V_{A,0} < D \Leftrightarrow p_G G + (1 - p_G)B < D \Leftrightarrow p_G < (D - B)/(G - B)$. Suppose that $B < D \leq G$. Then ability-to-pay solvency requires that $p_G \geq p_S$. The firm is balance-sheet insolvent and ability to pay solvent when $p_S < p_G < (D - B)/(G - B)$.

Example 1: Suppose $G = 100, B = 50, p_G = 0.6$, and $D = 85$. Then $V_{A,0} = 80, V_{D,0} = 71$, and $V_{E,0} = 9$. The debtor is balance-sheet insolvent, since $V_{A,0} = 80 < 85 = D$. If $p_S = 0.51$, that is, if the firm must be at least 51% likely to pay its debts in full to be considered ability-to-pay solvent, then the debtor firm is ability-to-pay solvent.

PROPOSITION 2: *Ability-to-pay insolvency does not imply balance-sheet insolvency. That is, $p_G I_{G \geq D} + (1 - p_G) I_{B \geq D} < p_S \nRightarrow V_{A,0} < D$.*

Proof: Suppose that $B < D \leq G$. Then the firm is ability-to-pay insolvent when $p_G < p_S$. The firm is balance-sheet solvent if $V_{A,0} \geq D \Leftrightarrow p_G \geq (D - B)/(G - B)$. The firm is then ability-to-pay insolvent but balance-sheet solvent when $(D - B)/(G - B) \leq p_G < p_S$.

Example 2: Suppose $G = 100, B = 50, p_G = 0.4$, and $D = 60$. Then $V_{A,0} = 70, V_{D,0} = 54$, and $V_{E,0} = 16$. The debtor is balance-sheet solvent, since $V_{A,0} = 70 > 60 = D$. If $p_S = 0.51$, that is, if the firm must be at least 51% likely to pay its debts in full to be considered ability-to-pay solvent, then the debtor firm is ability-to-pay insolvent, since there is only a 40% chance the firm will pay its debts in full, since the firm can pay its debts in full only in the good state, which has only a 40% chance of occurring.

Together, Propositions 1 and 2 show that the two solvency tests can give different indications of solvency and insolvency. Balance-sheet solvency compares the value of assets to the face value of debt. But asset value can increase in states of the world—the "good" state in the model—that have no effect on the expected value of the debt repayment if debt never receives the upside in such states. This highlights an objection to the balance sheet test: if viewed as a "measure" of solvency rather than a yes-no test, the measure can increase without any change in the value of the debt. This fits uneasily with the intuition that a "more solvent" debtor is one with a larger equity cushion.

In Example 2, for instance, there is a considerable surplus of cash in the good state, $G = 100$ while the amount required to pay debt in full is $D = 60$. This surplus drives up $V_{A,0}$, which is then compared to D, and yet that surplus is not available to pay debt in other states of the world and, in the model, does nothing to help the nonpayment of the debt in full in the bad state where $B = 50$ and there is thus a shortfall of 10 in paying the debt. Similarly, so long as there are collectively states of the world where the debt is payable in full, and the probability of those states occurring is at least p_S, the firm is ability-to-pay solvent. But that does not mean that the value of assets exceeds the face value of debt before the state of the world is revealed, and, in particular, debt will be left unpaid in other states to an extent that $V_{A,0} < D$.

C.   Debt Capacity

I define the firm's *debt capacity* at time $t = 0$ as $max_D V_{D,0}$, the maximum amount of debt that can be raised subject to constraints discussed below.[76] I start by examining debt capacity unconstrained by solvency tests. I then consider each of the ability-to-pay and balance-

---

[76] I follow the description of Stuart M. Turnbull. Stuart M. Turnbull, *Debt Capacity*, 34 J. FIN 931, 931 (1979) ("The existence of a maximum amount of credit that lenders are willing to extend to a firm provides a natural definition for debt capacity.").

sheet tests separately, and then (as in the real world) when the firm must satisfy both solvency tests.

PROPOSITION 3: *The ability-to-pay test constrains debt capacity.*

Proof: Absent any constraint from the solvency tests, debt capacity is maximized by choosing $D \geq G$
then $max_D V_{D,0} = p_G G + (1 - p_G)B = B + p_G(G - B)$, which is to say that the debt capacity is the cash flow in the bad state plus the expected incremental cash flow if the good state occurs. If the ability-to-pay test must be satisfied, then D must be chosen such that
$p_G I_{G \geq D} + (1 - p_G)I_{B \geq D} \geq p_S$. That is, there must be sufficient states of the world where the debt is repaid in full such that the total probability of one of those states occurring is at least p_S. Suppose that $p_G \geq p_S$. Then the firm can raise $D = G$ of debt, and the debt capacity is the same as the unconstrained debt capacity, $max_D V_{D,0} = p_G G + (1 - p_G)B = B + p_G(G - B)$. If $p_G < p_S$ then the firm can issue only $D = B$ of debt, and the firm's debt capacity is $max_D V_{D,0} = p_G B + (1 - p_G)B = B$.

Example 3: Without need to satisfy the ability-to-pay test, suppose $G = 100, B = 50$, and $p_G = 0.6$. Then $V_{A,0} = 70, V_{D,0} = 54$, and $V_{E,0} = 16$. $D = 60$. Then $V_{A,0} = 70, V_{D,0} = 54$, and $V_{E,0} = 16$. Setting $D = G = 100, max_D V_{D,0} = 80$ and $V_{E,0} = 0$. But if the ability-to-pay test must be satisfied and supposing that $p_S = 0.51$, then since $p_G > p_S$, we can again choose $D = G$, and $max_D V_{D,0} = 80$ and $V_{E,0} = 0$. This is the highest amount of debt that can satisfy the ability-to-pay constraint in this example where $p_G > p_S$. But if $G = 100, B = 50, p_G = 0.4$, and $p_S = 0.51$, then $V_{A,0} = 70$. Since $p_G < p_S$, the D that solves $max_D V_{D,0} = D = B = 50$. This gives $V_{E,0} = 20$.

PROPOSITION 4: *The balance-sheet test constrains debt capacity.*

Proof: Absent any solvency constraint, debt capacity is maximized by choosing $D \geq G$. Then $max_D V_{D,0} = p_G G + (1 - p_G)B = B + p_G(G - B)$, which is to say that the debt capacity is the cash flow in the bad state plus the expected incremental cash flow if the good state occurs. If the balance-sheet test must be satisfied, then the firm can only issue an amount of debt that leaves it balance-sheet solvent after borrowing. That is, $p_G G + (1 - p_G)B \geq D \Leftrightarrow D \leq B + p_G(G - B)$. If we set the constraint to be binding, then $D = B + p_G(G - B)$. Since $B + p_G(G - B) \Leftrightarrow G - (1 - p_G)G + (1 - p_G)B$, which is less than G because $(1 - p_G)G > (1 - p_G)B$, this means that $D < G$. Therefore, $max_D V_{D,0} = p_G min\{G, D\} + (1 - p_G)min\{B, D\} = p_G min\{G, B + p_G(G - B)\} + (1 - p_G)min\{B, B + p_G(G - B)\} = p_G(B + p_G(G - B)) + (1 - p_G)B = B + p_G^2(G - B) < B + p_G(G - B)$.

Example 4: Suppose $G = 100, B = 50$, and $p_G = 0.6$. Then $V_{A,0} = 80$. The D that solves $max_D V_{D,0}$ is $D = B + p_G(G - B) = 50 + 0.6(100 - 50) = 80$. This gives $max_D V_{D,0} = 68$ and $V_{E,0} = 12$. This is the highest amount of debt that can satisfy the balance-sheet test since $V_{A,0} = D = 80$.

If both solvency tests must be satisfied, then it is clear from Propositions 3 and 4 that the debt capacity of the firm is B if $p_G < p_S$ (since in that case the ability-to-pay test is binding, even though the balance-sheet test is slack) and $B + p_G^2(G - B)$ if $p_G > p_S$ (since in that case the balance-sheet test is binding and the ability-to-pay test is slack).

Example 5: Suppose $G = 100, B = 50, p_G = 0.4$ and $p_S = 0.51$. Then $V_{A,0} = 70$ and $D = B = 50$. This gives $max_D V_{D,0} = 50$, and $V_{E,0} = 20$. This satisfies both the ability-to- pay solvency test (because the firm can pay B in all states (100% of the time) and so more than 51% of the time) and the balance-sheet test, since $V_{A,0} = 70 > 50 = D$. If $G = 100, B = 50, p_G = 0.6$ and $p_S = 0.51$, then $V_{A,0} = 80$ and $D = 80$. This gives $max_D V_{D,0} = 68$ and $V_{E,0} = 12$. This satisfies the ability-to-pay test since $G > D$ and $p_G > p_S$ and satisfies the balance-sheet test since $V_{A,0} = D = 80$.

Recall that the value of the assets at time $t = 0$ is the probability-weighted sum of the good and bad cash flows, $V_{A,0} = p_G G + (1 - p_G)B = B + p_G(G - B)$. But this is the same as the unconstrained debt capacity $max_D V_{D,0} = p_G G + (1 - p_G)B = B + p_G(G - B)$. Therefore, without the solvency constraint, the firm can raise 100% of its asset value as debt (we ignore the impact of tax laws on the characterization of a firm with 100% debt).

Once we add the solvency test constraints, however, debt capacity falls as a percentage of asset value. If debt capacity is B because $p_G < p_S$ and the ability-to-pay test is binding, then debt capacity as a percentage of asset value falls to $B/[B + p_G(G - B)]$. If debt capacity is $B + p_G^2(G - B)$ because $p_G \geq p_S$ and the balance-sheet test is binding, then debt capacity as a percentage of assets increases to $[B + p_G^2(G - B)]/[B + p_G(G - B)]$, still below the unconstrained case.

Figure 1 demonstrates the change in debt capacity as a percentage of asset value. For values of p_G from 0 to 1 where $G/B = 2$, the balance-sheet test is binding, and the ability-to-pay test is binding until $p_G \geq p_S = 0.51$.

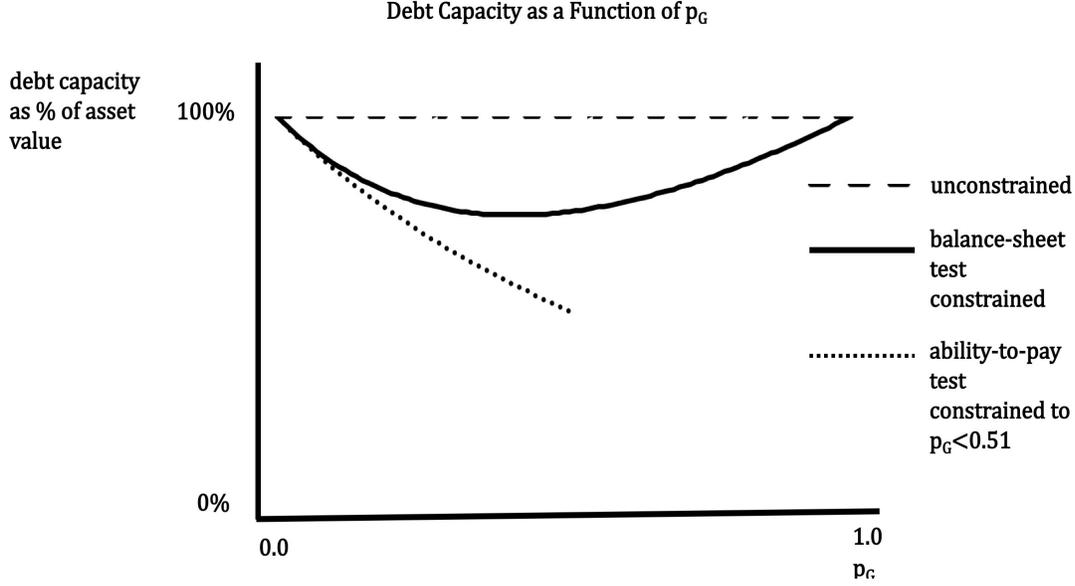Debt Capacity as a Function of p_G

Figure 1: *Maximum debt capacity as a percentage of asset value for unconstrained borrowing and borrowing constrained by the balance-sheet test and the ability-to-pay test to $p_G \geq p_S = 0.51$.*

D. Bounds on Promised Yields

I next show how legal insolvency limits the yield that firms can promise new creditors.

PROPOSITION 5: *The ability-to-pay and balance-sheet tests limit the yields the firm can promise on its debt.*

Proof: Without constraint, $D = G$, so the promised interest over the borrowing period is $G/[B + p_G(G - B)] - 1$. In the case of constrained borrowing where debt capacity is B because $p_G < p_S$ and the ability-to-pay test is binding, then $D = G$ and the promised interest over the borrowing period is $B/B - 1$, which is 0, and therefore smaller than the unconstrained promised interest since $G > B + p_G(G - B) = p_G G + (1 - p_G)B$, so long as $G > B$ and $p_G > 0$. In the case of constrained borrowing where debt capacity is $B + p_G^2(G - B)$ because $p_G \geq p_S$ and it is the balance-sheet test that is binding, then $D = B + p_G(G - B)$ and the promised interest over the borrowing period is $[B + p_G(G - B)]/[B + p_G^2(G - B)] - 1$, which is greater than 0 but less than the unconstrained promised interest, $G/[B + p_G(G - B)]$. This is true because $[B + p_G(G - B)]/[B + p_G^2(G - B)] < G/[B + p_G(G - B)] \Leftrightarrow [B + p_G(G - B)] - [B + p_G^2(G - B)] < G - [B + p_G(G - B)] \Leftrightarrow -p_G^2 < [1 - 2p_G] \Leftrightarrow 0 < p_G^2 - 2p_G + 1 \Leftrightarrow 0 < (p_G - 1)(p_G - 1)$, which is true because the p_G are probabilities assumed greater than zero and thus both factors are negative.

15

Example 6: With no constraints from legal solvency tests, suppose $G = 100, B = 50, p_G = 0.6$. Then $V_{A,0} = 80$. Setting $D = G$, $max_D V_{D,0} = 80$, and the promised interest over the borrowing period is $100/[50 + 0.6(100 - 50) - 1 = 0.25$ or 25%. With legal solvency tests, $D = 80$ so $max_D V_{D,0} = 68$ and the promised interest over the borrowing period is $80/68 - 1 = 0.176$ or 17.6%. Suppose instead that $G = 100, B = 50, p_G = 0.4$ and $p_S = 0.51$. Setting $D = G$, $max_D V_{D,0} = 70$, and the promised interest over the borrowing period is $100/[50 + 0.4(100 - 50) - 1 = 0.25$ or 25%. Then from above, $V_{A,0} = 70$ and $D = B = 50$ so $max_D V_{D,0} = 50$ and the promised interest over the borrowing period is $50/50 - 1 = 0.0$ or 0%.

Figure 2 demonstrates the change in maximum promised yields for values of p_G from 0 to 1 where $G/B = 2$ and the balance-sheet test is binding.[77]
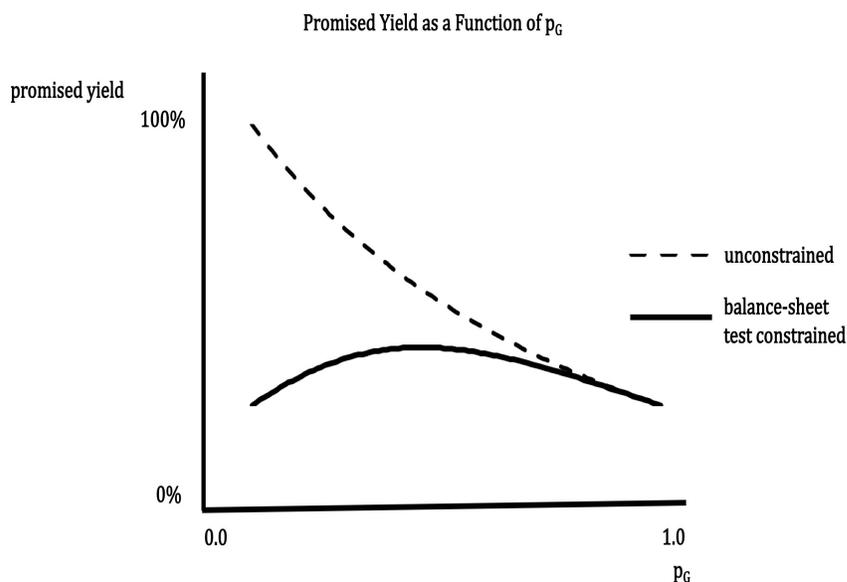

Figure 2: *Maximum promised yield for unconstrained borrowing and borrowing constrained by the balance-sheet test.*

IV.   *The Black-Scholes Model of an Issuing Firm*

Black and Scholes[78] and Merton[79] pointed out that in a firm with debt outstanding, the firm's equity can be analyzed as a call option on the firm's underlying assets with the amount of

---

[77] The promised yield is 0 when the ability-to-pay solvency test is binding.
[78] Fischer Black & Myron Scholes, *The Pricing of Options and Corporate Liabilities*, 81 J. POL. ECON. 637 (1973).
  A.   [79] Robert C. Merton, *On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 29

  J. FIN. 449, 454 (1974) (analyzing the isomorphic relationship between equity, call

  options, and debt).

the debt repayment obligation at the debt's maturity as the call option's strike price. I now redo the article's results in the Black-Scholes framework to establish their generality.

A. Model Set Up

We start with the familiar Black-Scholes call option pricing formula[80] but in terms applicable to analysis of firm equity, assets and debt:

$$V_{E,t} = N(d_1)V_{A,t} - N(d_2)De^{-r(T-t)}$$

where $d_1 = \frac{1}{\sigma\sqrt{T-t}}[\ln\left(\frac{V_{A,t}}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)]$ and $d_2 = d_1 - \sigma\sqrt{T-t}$.

In this representation, we use $V_{E,t}$ to denote the market value of the firm's equity at time t, $V_{A,t}$ to denote the market value of the firm's assets at time t, and D to denote the *face value* of the firm's debt that matures at time T.[81] As in the standard representation, r is the annual risk-free rate with continuous compounding, σ is the volatility of returns of the firm's assets, and $N(.)$ is the cumulative distribution function of the standard normal distribution, that is, the probability that the value within the parentheses will be at or below that amount if drawn from a normal probability distribution with mean of 0 and standard deviation of 1.[82]

I analyze the firm at initial debt issuance, time $t = 0$. The market value of the assets at time $t = 0$ is $V_{A,0}$. Debt is entitled to full payment of D or the entire cash flow in the state, whichever is smaller. The value of the debt at time $t = 0$ is $V_{D,0}$. As above, the debtor firm is *balance-sheet solvent* at time $t = 0$ if and only if $V_{A,0} \geq D$. The firm is thus *balance-sheet insolvent* if $V_{A,0} < D$. In the Black-Scholes context, we can approximate our ability-to-pay solvency test by defining the firm to be *ability-to-pay solvent* at time $t = 0$ if and only if $N(d_1) \geq p_S$, where $N(d_1)$ is approximately the probability that the firm's asset value exceeds its debt at time T.

B. Debt Capacity

The results for debt capacity are straightforward in the Black-Scholes framework.[83]

PROPOSITION 6: *The ability-to-pay test constrains debt capacity.*

Proof: Absent a solvency-test constraint, debt capacity is maximized by choosing an arbitrarily large D such that all asset value at time T is paid out to debt holders so that $N(d_1)$ is arbitrarily close to zero. But $N(d_1)$ is arbitrarily close to zero means that $N(d_1) < p_S$ so long as p_S is not also arbitrarily close to zero (which would be equivalent to the absence of an ability-to-pay test). Therefore, ability-to-pay solvency requires a smaller D than in the unconstrained case.

PROPOSITION 7: *The balance-sheet test constrains debt capacity.*

---

[80] Black & Scholes, *supra* note 78, at 644, Eq. 13.
[81] I consider a zero-coupon bond here so that concern is with "solvency risk" of being unable to pay the debt at maturity, and not what some authors have called "liquidity risk" concerning the ability to pay pre-maturity coupons. *See*, e.g., Sebastian Gryglewicz, *A Theory of Corporate Financial Decisions with Liquidity and Solvency Concerns*, 99 J. FIN. ECON. 365, 366 (2011).
[82] Black & Scholes, *supra* note 78, at 644.
[83] *See generally* Black & Scholes, *supra* note 78.

Proof: Absent a solvency-test constraint, debt capacity is maximized by choosing an arbitrarily large D such that all asset value at time T is paid out to debt holders so that $N(d_1)$ is arbitrarily close to zero. But this means that $D \gg V_{A,0}$ and the firm is insolvent. Thus, balance-sheet-solvency requires smaller D than in the unconstrained case.

C.   Bounds on Promised Yields

It also is straightforward in the Black-Scholes framework to show that the legal solvency tests bound promised yields.[84]

PROPOSITION 8: *The ability-to-pay test and balance-sheet tests limit the yields that the firm can promise on its debt.*

Proof: Without the legal solvency tests, the firm can promise a yield of $V_{A,T}/V_{A,0}$ by setting D arbitrarily large so that the final asset value $V_{A,T}$ will be paid in full to the creditors. But to satisfy the ability-to-pay solvency test, the firm must set D small enough that $N(d_1) \geq p_S$. This means that the ability-to-pay test limits promised yields. To satisfy the balance-sheet test, the firm can set $D = V_{A,0}$ at most, which must be lower than the arbitrarily large amount promised in the unconstrained world. This means that the balance-sheet test limits promised yields as well.

V.   *Conclusion*

Research on security design that ignores legal restrictions is unlikely to shed much light on real-world behavior. In this article, I explore two theoretical results about debt and firms that take seriously one of the most important legal impacts on debt: legal insolvency. I derive these results from the premise that firms must avoid legal insolvency when issuing new debt because insolvency at issuance would trigger severe operational limitations on the firm, including the ability of creditors to enforce fiduciary duties and legal restrictions on managerial decisions that otherwise would have the protections of the business judgment rule. The article's results demonstrate that legal insolvency probably has important – if largely unnoticed – effects in limiting debt capacity and debt yields. More generally, the article illustrates the usefulness of better understanding legal solvency requirements, a legal mechanism that has received too little attention in the understanding of debt and security design.

---

[84] *See generally* Black & Scholes, *supra* note 78.